# Mask Design for Covid-19

Neville D. Fowkes [1]†

[1] *University of Western Australia, W.A., Australia*

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**Communicated by:** David P. Mason, University of the Witwatersrand, Johannesburg

**Industrial Partner:** South African Centre for Epidemiological Modelling and Analysis, University of Stallenbrosch.

**Presenter:** Alex Welte, South African Centre for Epidemiological Modelling and Analysis, University of Stallenbrosch.

**Team Members:** Precious Chiwira; AIMS, South Africa; Thato Magodi, AIMS, South Africa; Fameno Rakotoniaina, AIMS, South Africa.

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**Summary**

Simple Bernoulli and air permeability models are used to estimate the pressure developed inside a rigid covid-19 mask and the proportion of air flow passing through and leaking around the edges of the mask due to a sneeze or cough. Rather speculatively the results are extended to deal with the commonly used cloth masks with or without folds. This is done by introducing a constitutive law connecting the pressure developed within the space between the mask and the face and the volume of this space. The model identifies a dimensionless group and a design function that may be used to help evaluate the performance of masks in the covid-19 context. These measures may be simply measured in the laboratory.

† Corresponding Author: `neville.fowkes@uwa.edu.au`

## 1 Introduction

It has been found that even with specially designed well fitting masks significant leakage occurs around the mask edges [2], so covid-19 virus bearing particles will escape. One such commonly used 'particulate filter respirator' is N95. Whilst such well designed masks may be efficient at removing virus bearing particles they are usually uncomfortable to wear with wearers having difficulty breathing, and individuals with lung conditions may be advised to not to use such masks. Evidently a crude hand made cloth mask is likely to be more comfortable but less efficient in terms of preventing virus spread; a compromise in design is needed here.

There are many different mask types (textile, surgical, respiratory) using different filters and structural design. The filters may be made up of laid down fibres or woven cloth. The cloth masks normally used to reduce covid-19 spread may or may be woven, may or may not contain folds, and may have a structurally supported breathing space. The problem of determining the flow through and around a mask due to a sneeze is a major computational task that would need to be tailored to the specifics of the mask. Such a calculation may be useful if the aim is to design a specific mask, but that is not our aim here. Our aim here is to identify general design features that may be used to assess the performance of all masks in the covid-19 context. These features need to be measurable using simple experiments and such features should provide general guidance concerning design.

In Section 2 we develop the flow model. We then discuss mask design using the rigid mask as a datum and then go on to discuss the use of a constitutive law model to describe the behaviour of cloth models with or without folds or added breathing space, see Section 3. In Section 4 we determine the behaviour of masks of different quality and design under sneeze or cough forcing. Finally, in Section 5 we draw conclusions concerning the validity and usefulness of the proposed model.

## 2 The Defining Equations

Masks fit reasonably snugly on the face but there will be a space (volume $V$) between the face and the mask, and there will be an effective separation distance $\delta$ around the mask edge, see Figure 1. The effect of a cough or a sneeze will be to cause an increase in air pressure within this space. We will denote this increase in pressure above that of the environment (assumed to be at atmospheric pressure, which is chosen as a datum) by $p$. This pressure increase will cause an increase in volume $V$ within this space and also result in an air flow through the face of the mask (volume flow rate denoted by $Q_m$) and a leakage flow from around the sides of the mask (volume flow rate denoted by $Q_l$). Our concern is with the dependence of these flows on the physical parameters of the mask and parameters determining the snugness of fit to the face.

### 2.1 A Bernoulli Model of leakage flow

The steady state Bernoulli's equation gives

$$p + 1/2\rho v^2 = c, \tag{2.1}$$

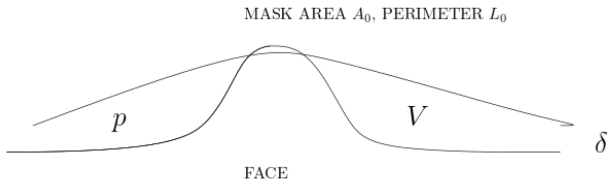


Figure 1. Mask geometry: The mask (area $A_0$) fits onto the face. The volume of air between the face and the mask is $V$. The pressure within this space above atmospheric pressure is $p$. The effect of a sneeze or cough will will be to increase $p$ and $V$. The effective spacing around the edge of the mask with perimeter $L_0$ is $\delta$.

where $\rho$ is the density of air, $v$ is the velocity of a fluid particle, and $c$ is a constant for the element. This gives an expression for the leakage velocity $v_l$ of air particles escaping from the mask space through the sides as a result of a internal pressure increase of $p$ as

$$v_l = \sqrt{2p/\rho}, \tag{2.2}$$

with the associated total leakage volume flow rate through the gap area $L_0\delta$ given by

$$Q_l = \alpha\sqrt{2p/\rho}(L_0\delta), \tag{2.3}$$

where $\alpha$ is the coefficient of contraction[1].

*The steady state Bernoulli approximation*

The above Bernoulli equation approximation is central to the leakage model and so requires some justification. It is in a real sense 'a quick fix', enabling us to obtain crude results without detailed calculations. In general terms to improve this approximation one would need to solve the full Navier-Stokes equations with the mask geometry unknown and to be determined as part of the calculation. Also it would be necessary to detail the inflow due to the sneeze. Of course detailed data would be needed to support such a calculation.

The steady state Bernoulli equation is derived from the momentum conservation equation under steady, incompressible and inviscid flow conditions and is an energy conservation statement for a particle of fluid escaping the mask space through the sides. The contraction coefficient $\alpha$ is primarily introduced to account for the contraction of streamlines as they pass around the edges of the mask, resulting in a change in the effective area of the gap (a vena contracta effect [1]). However other inadequacies in the Bernouilli model are absorbed into this experimentally determined coefficient. The Reynolds number of the flow is about 4400 [2], so the flow is essentially inviscid. Also the pressure changes within the mask are small compared with atmospheric pressure, so the flow is essentially incompressible. Typically for sharp orifices the contraction coefficient is taken as $\alpha = 0.61$, and although this correction factor is not close to unity it has been found

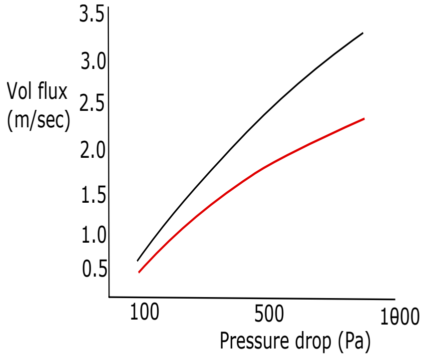


Figure 2. Air permeability data: *Left:* Volumetric flux (m/sec) of two non-weave sample fabrics as a function of pressure drop across the fabric. *Right*: Changes in the air permeability in a woven fabric due to weave spacing. The upper curve has a weft number of to 60, the lower a weft number of 20. In both cases the warp number is 20 Nm, with the number of warp yarns per cm: 20; data taken from Ogulata [6].

to not vary greatly with flow circumstances; evidently it is the abrupt change in flow geometry at sharp edges that dominates.

The steady state assumption: The time span for air particles to traverse the mask space[1] is generally small (approximately 4.0 $10^{-2}$ secs) compared with the mask inflation time (approximately 0.15 secs, see later), so the steady state assumption seems justified.

The primary error in the above calculation however is likely to arise as a result of the use of an effective gap size to describe the fit of the mask to the face. Usually the gap size will be greater either near the nose crease or the cheeks and, depending on the mask type, may change when sneezing or coughing.

In general terms the Bernoulli model can be thought of as a simple but crude engineering model which contains the most important features of the process, but requires fitting.

### 2.2 Flow through the mask: air permeability

In the textile industry the air permeability $k_{\mathrm{air}}$ (m/(Pa sec)) is defined by

$$q = k_{\mathrm{air}} p, \tag{2.4}$$

where $q$ is the volume flux (m/sec) through of the fabric driven by a pressure difference $p$ (Pa) across the fabric. Typical experimental plots of volume flux vs pressure drop for a fabric are shown in Figure 2 *Left*. For a particular fabric the slopes of these curves remains constant over pressure ranges of interest so the air permeability of a fabric is well defined.

[1] based on a typical flow speed of 5 m/sec for a cough [2], and a mask perimeter of 20 cm

Of course the mask filter may be made of one or several layers of fabric (typically three[2]) in series; the net air permeability is obtained by addition.

The total volume flow rate through the mask face of area $A_0$ is

$$Q_m = k_{\text{air}} p A_0. \tag{2.5}$$

### 2.3 Conservation of mass, sneeze/cough input

Mass conservation in the mask space requires

$$\frac{dV}{dt} = Q_{in} - [Q_l + Q_m], \tag{2.6}$$

where $Q_{\text{in}}(t)$ ($\text{m}^3$/sec) is the volume flow rate into the mask space due to a cough or sneeze.

If we assume an equation of state $V(p)$ can be used to determine the behaviour of the mask under inflation then this equation reduces to

$$[\frac{dV}{dp}]\frac{dp}{dt} = Q_{in}(t) - [Q_l(p) + Q_m(p)], \tag{2.7}$$

where we've explicitly noted the dependence of mask and leakage fluxes on the pressure $p(t)$ within the mask air gap. The equation of state is determined by the geometry, structure and composition of the mask, the elasticity of the support, as well as the inflation pressure.

Equation 2.7 is an ordinary differential equation for determining $p(t)$, which can be solved for a prescribed volumetric flow input rate $Q_{in}(t)$. The associated mask and leakage flow rates can then be determined using (2.5, 2.3).

### 2.4 Scaling

We introduce scales so as to reduce the flux equation (2.7) to its simplest form and identify the important dimensionless groups. We write

$$Q_{in} = \bar{Q} Q'_{in}(t'), Q_l = \bar{Q} Q'_l(t'),\ t = t_0 t', p = p_0 p', v = V_0 V', \tag{2.8}$$

where $\bar{Q}$ is a typical air flux ($\text{m}^3$/sec) from the nose or mouth, $V_0$ a typical mask space volume, and we choose

$$t_0 = \frac{V_0}{\bar{Q}},\ p_0 = \frac{\bar{Q}}{k_{\text{air}} A_0}; \tag{2.9}$$

the inflation time scale and the inflation pressure associated with the flux input $\bar{Q}$. With this choice the flux equation reduces to its simplest dimensionless form:

$$[\frac{dV'}{dp'}]\frac{dp'}{dt'} = Q'_{in} - [\xi\sqrt{p'} + p'], \tag{2.10}$$

where we will refer to the dimensionless group

[2] Some masks have a water repellent outer layer, a middle layer designed to filter out particles and an inner moisture absorbing layer.

$$\xi = [\frac{\alpha}{k_{\text{air}}}\sqrt{\frac{2}{(\rho p_0)}}][\frac{L_0\delta}{A_0}] \equiv [\mathcal{V}_r][\mathcal{A}_r]], \tag{2.11}$$

as the *quality parameter*, and where we have separated out the velocity ratio and the area ratios components associated with this parameter. As suggested by the naming, $\xi$ provides a measure for the ratio of the leakage flux to the mask flux; a well fitted mask with a fine filter corresponds to a *smaller* value of $\xi$.

Dropping primes we get

$$[\frac{dV}{dp}]\frac{dp}{dt} = Q_{in} - [\xi\sqrt{p} + p]. \tag{2.12}$$

It should be noted that with our simple model *just two factors*, the *quality parameter* $\xi$, and the *design function* $[\frac{dV}{dp}]$, are needed to characterise the air exchange behaviour of masks associated with a volume flow rate input.

## 2.5 Data

*Mask and cough/sneeze data*

Much of this data presented here is taken from Dbouk and Drikakis [2]. The typical cloth mask covers the nose and mouth and is approximately of size 20 cm by 12 cm, with a gap around the sides of mask varying from a minimum of 4 to 6 mm to a maximum of 1.4 cm (the nose to eye corner). If one assumes an average face to mask gap of 6 mm then the volume of space between the mask and face is given approximately by $V_0 = 9.3\ 10^{-4}$ m$^3$. The mouth area under coughing conditions is typically 1.937 cm$^2$ and flow velocities of 5 m/sec are to be expected [2]. Based on these values we get a mask inflation time scale of $t_0 \approx 0.15$ sec. The estimated time span of a cough is 0.12 secs, which is about the same as the mask inflation time scale.

The leakage model above indicates a typical velocity of $\alpha\sqrt{\frac{2p}{\rho}}$, which for a (typical) pressure difference of 100 Pa gives 7.7 m/sec, compared with the quoted value of 5 m/sec) in Dbouk and Drikakis [2].

*Air permeability data*

The air permeability of the filter is influenced by the fabric's material and structural properties, such as the raw material of the fabric, whether or not it is non-woven (wetlaid, melt or spun) or woven. If woven, the spacing between weaves, the particular weave, the set of yarns, yarn twist, cloth treatment and finishing, also effect the permeability, see [5], [6].

Typical values for non-woven cloth and woven cloth are displayed in Figure 2. Based on these figures we get values of air permeability of (2.6 to 3.8) $10^{-3}$ m/(sec Pa) for non-weave samples, and (0.2 to 1.2) $10^{-3}$ m/(sec Pa) for the woven samples. The associated through-flow velocities due to a (typical) pressure drop of 100 Pa are (2.6 to 3.8) m/sec for non-woven materials (corresponding to sample 3 and sample 2 in the figure) and (0.2 to 1.2) m/sec for woven materials (bottom curve and top curve in the figure).

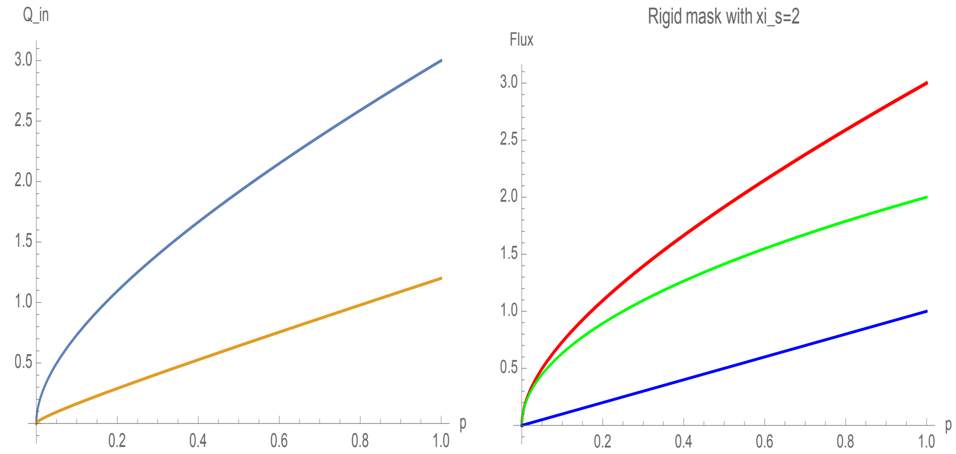


Figure 3. Rigid mask behaviour under inflation: *Left* Behaviour of solid masks with different $\xi$'s under inflation: Top curve $\xi_s = 2$ (a leaky mask), Bottom curve $\xi = 0.2$ (a well fitted mask) *Right* Flux apportionment as a function of inflation pressure $p$: Flux input $Q_{in}$ (top, red), leakage flux component $Q_l(p)$ (middle, green), mask flux $Q_m(p)$ (lower, blue)

*The quality parameter $\xi$*

Recall that the quality parameter $\xi$ is the product of the area ratio $\mathcal{A}_r$ times the velocity ratio parameters $\mathcal{V}_r$. The area ratio is typically small ($\delta$ being relatively small) and the velocity ratio typically large, again because $\delta$ is small, so the product can be either small or large depending on the mask filter and fit. Based of the above data we get:

- for woven filters $\xi$ ranges from 0.64 to 7.6,
- for non-woven filters $\xi$ ranges from 0.26 to 0.7.

You will recall that small $\xi$ values correspond to quality masks, so we can see that non-woven masks are generally of higher quality, basically because the pore size is smaller for these masks. These masks are better able to remove small particles but higher pressure differences are needed to drive the through-flow. Of course these masks are also less comfortable. Whilst the above values are representative and seem reasonable, it should be pointed out that the face fitting gap can be 2 mm for a well fitted mask to 1.5 cm for a crude fit, and the permeability can also vary over several orders of magnitude, so that much smaller and larger values of the quality parameter are possible.

For the simulations to follow we will use values $\xi = 2$ (a poor quality mask), and $\xi = 0.2$ (a good quality mask).

## 3 Mask design

The static behaviour of the mask under inflation is described by the state equation $V(p)$, however it is *mask design function* $[\frac{dV}{dp}]$ that determines the mask's dynamical behaviour.

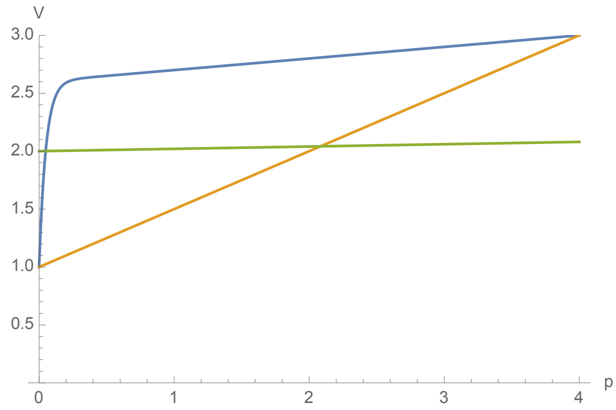


Figure 4. State equations for various mask types (illustrations only): Upper (blue) curve, a cloth mask with folds $(a, b, c, d) = (2.6, 0.1, 1.6, 0.05)$. Middle (green) curve, a rigid mask $(a, b, c, d) = (2, 0, 0, 1)$. Lower (yellow) curve, a cloth mask without folds $(a, b, c, d) = (1, 0.5, 0, 1)$.

*Rigid design masks*

If the mask is rigid then no change in the mask volume space $V$ occurs during a sneeze or cough, so that in this case $\frac{dV}{dt} = 0$, and (2.12) gives

$$Q_{in} = \xi\sqrt{p} + p; \tag{3.1}$$

a quadratic in $\sqrt{p}$ which determines the pressure $p$ due to any prescribed flux input $Q_{in}(t)$. The associated leakage and mask fluxes can be recovered using (2.3),(2.5 ).

Note that if $p$ is small in (3.1) then the leakage flux term $\xi\sqrt{p}$ dominates (and increases rapidly with $p$), but for larger values of p the linear mask through-flow term takes over so filter efficiency increases, see Figure 3.

*Non-rigid mask designs: the state equation $V(p)$*

Simple cloth masks expand uniformly under increasing pressure, whereas cloth masks with folds expand rapidly (with folds unfolding) with increasing pressure until the mask unfolds and then the expansion rate is slow. These situations can be modelled using a state equation of the form

$$V(p) = a + bp + ce^{-\frac{p}{d}} \tag{3.2}$$

and appropriately choosing $(a, b, c, d)$, to fit experimental results for masks. No such data has been collected to the author's knowledge, so we will work with artificial data. The state equation models with associated parameter values we will use are displayed in Figure 4. The rigid mask state equation is also included.

*Discussion*

The introduction of a state equation assumes that such exits and can sensibly be used to determine the through-flow and leakage fluxes. It could be that the various mask components simply act inertially, that is that the effect of the impulsive sneeze jet is to cause the mask components to instantaneous (and independently) move with a speed determined by the local impulse. Subsequently the various components may come to rest due to local mechanical damping caused by friction between cloth fibres. Eventually, however, under inflation the global constraining elastic forces will take over and the movement will be more predictable. For example in the folded mask case the initial unfolding process will (in detail) not be measurable, let alone determinable. The associated volume change within the mask will be predictable, but still until global elastic restoring forces take effect the result will not be determinable in terms of $p$. Fortunately from our point of view the determination of the initial mask movement is not of interest because there is little air exchange (and particle dispersal) during this stage. We therefore expect the results obtained for particle dispersal using the equation of state model to be reasonably accurate.

## 4 Mask Response Curves for a Sneeze or Cough

We model a sneeze or cough as a unit flux input over a unit time interval, corresponding to an un-scaled volume input rate of $\bar{Q}$ over an un-scaled time interval of $t_0 = 0.12$ secs. With this input we determine the response for masks of different quality ($\xi = 2$, $\xi = 0.2$) and design as determined by the state equations $V(p)$ displayed in Figure 4.

### *The rigid mask response*

Here we look at the rigid mask response to a sneeze input, see Figure 5. Because the mask is rigid and air is incompressible, the pressure response to the input flux is immediate, and the apportionment of leakage flux to mask through-flow is as described earlier and determined by the quality parameter $\xi$. Also immediately after the sneeze the mask deflates (with this simple model). The pressure buildup within the mask is greater for the high quality mask ($\xi = 0.2$), and the mask flow is also greater.

### *The cloth mask response*

For the cloth mask without folds, see Figure 6, the mask space pressure initially increases rapidly and after the sneeze finishes the pressure relatively slowly decays to zero for the parameter values chosen. As indicated earlier much of the initial flux leaks out the sides of the mask for both the low and high quality masks. At higher pressures much of the flux from the sneeze leaks out the sides for a low quality mask, whereas it passes through the mask if $\xi$ is small. As one would expect for the high quality mask it takes significant time for the mask to deflate after a sneeze. This is especially important if there are repeated sneezes/coughs.

For the cloth mask with folds the pressure build up within the mask and associated fluxes are small until the folds unfold, and the deflation of the mask is slow, see Figure 7. Essentially this mask behaves like an almost rigid mask with greater initial mask space. From the comfort point of view this initial space should be chosen to accommodate the expelled volume from normal breathing. In all the cases displayed the folds are such that

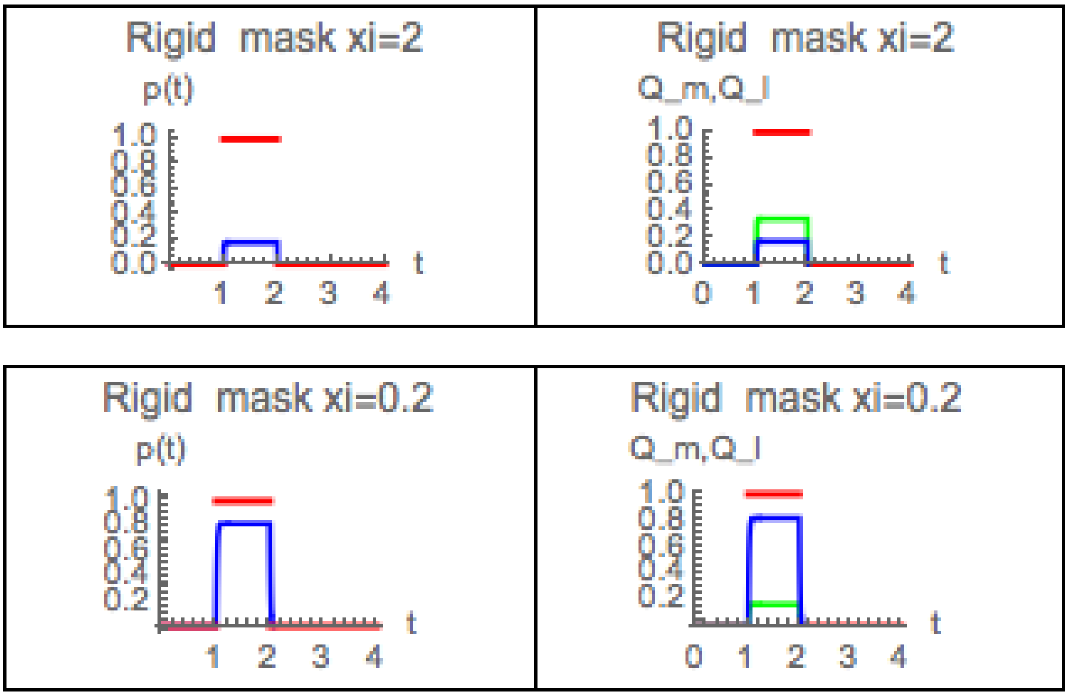


Figure 5. Rigid mask plots: The top figures correspond to a ($\xi = 2$) low quality mask, the bottom figures to a high quality ($\xi = 0.2$) mask. The left hand figures show $p(t)$ (blue), and for reference we have also displayed $Q_{in}(t)$ (red) (not to scale). The right hand figures display the input flux $Q_{in}$ (red), the leakage flux $Q_l$ (green) and the membrane flux $Q_m$ (blue).

there is little flow through the mask or leakage flow but of course for a more intense sneeze this would not be the case.

## 5 Summary

The above model is a simple engineering style model, and herein lies it's potential value. It should be a straight forward matter to experimentally determine the quality parameter and design function for a mask, and the volumetric response to a sneeze or cough should be relatively easy to measure. There are a number of underlying assumptions in the model, so the results certainly need to be checked out experimentally. Most notably the Bernoulli model contains the contraction coefficient $\alpha$ which needs to be fitted. For the model to be useful $\alpha$ needs to vary by little for different masks. As indicated in the text any further sophistication in either the fluids description or the mask response to forcing would require a much more detailed model.

In the above work we have only looked at the flow induced by a sneeze or cough. The work can be extended in an obvious way to deal with the periodic input due to a cough spasm. Additionally one can model a comfort index associated with a mask; some combination of the pressure level within the mask and the duration time of the sneeze

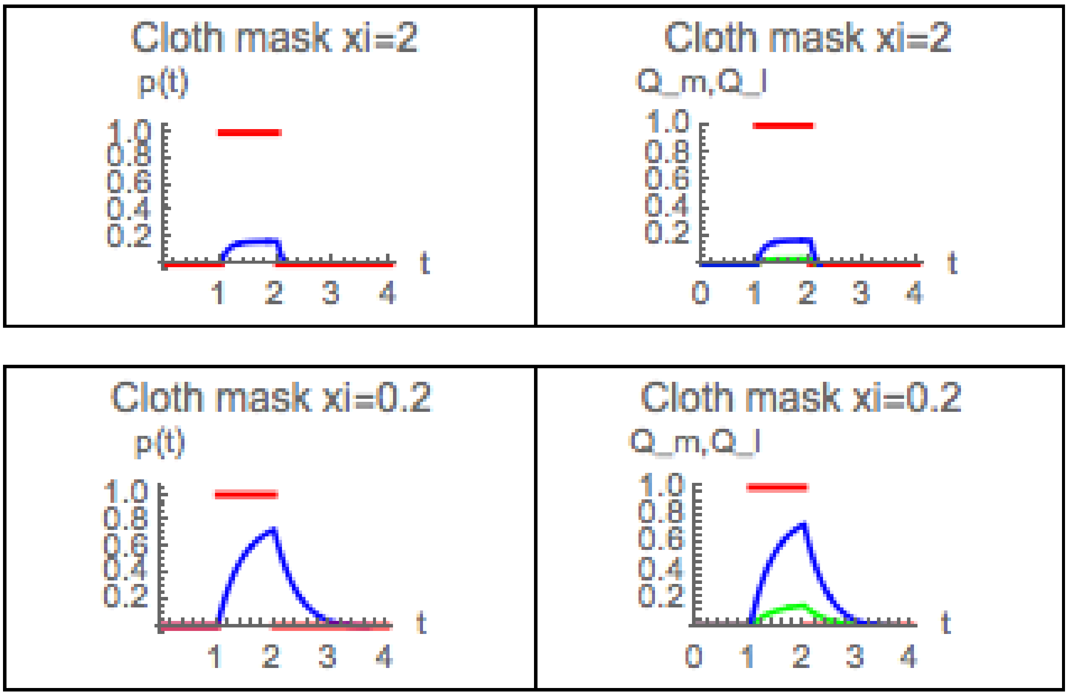


Figure 6. Cloth mask plots: The top figures correspond to a ($\xi = 2$) low quality mask, the bottom figures to a high quality ($\xi = 0.2$) mask. The left hand figures show $p(t)$ (blue), and for reference we have also displayed $Q_{in}(t)$ (red) (not to scale). The right hand figures display the input flux $Q_{in}$ (red), the leakage flux $Q_l$ (green) and the membrane flux $Q_m$ (blue).

could be used; experimental input will be essential here. The implications in terms of the capture of particles released by a sneeze or cough and the leakage of particles is yet to be explored. A simple model would assume that all particles in the through-flow stream are captured by the mask whilst all others are released into the environment. A better model would take into account the size distribution of particles released by the sneeze and the pore size in the filter.

In general terms the above work suggests that a rigid mask with an initial mask volume compatible with the volume of air released with a sneeze or a cloth mask with the same unfolded volume are sensible designs for comfort. None of this is unexpected but the above model provides a practical means for quantifying the quality of different masks and the expected response to a cough or sneeze.

We should remind the reader that the state equations chosen here were illustrative and not based on real mask data and the results were only obtained for one representative sneeze intensity. Much more work will be necessary to explore the model but first some experimental confirmation should be sought.

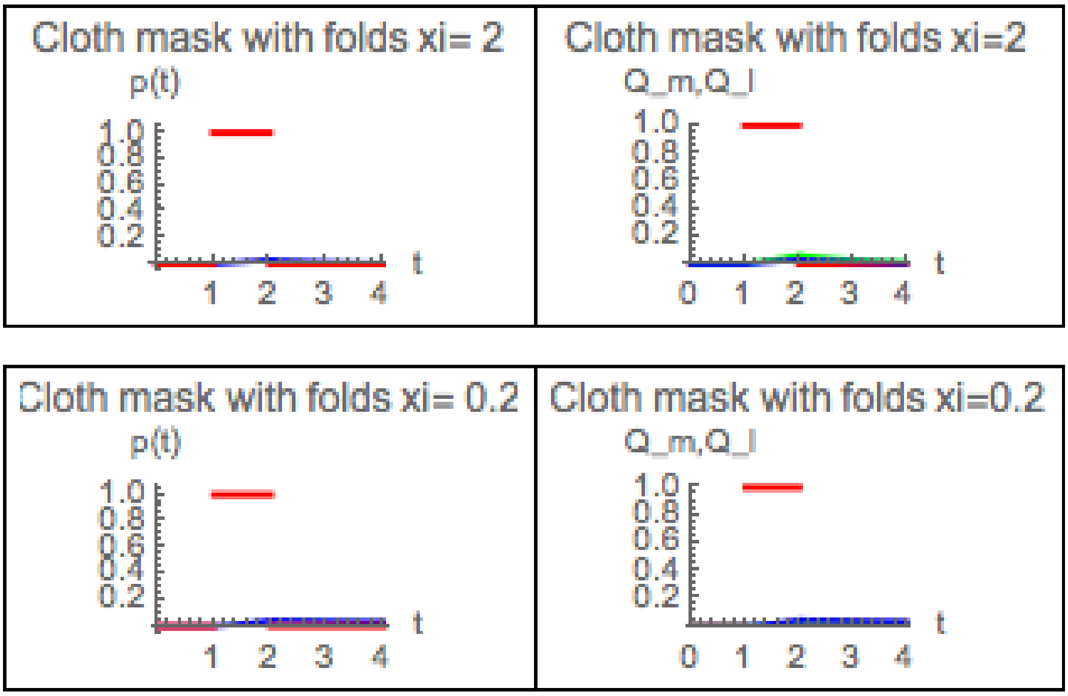


Figure 7. Cloth mask with folds: The top figures correspond to a ($\xi = 2$) low quality mask, the bottom figures to a high quality ($\xi = 0.2$) mask. The left hand figures show $p(t)$ (blue), and for reference we have also displayed $Q_{in}(t)$ (red) (not to scale). The right hand figures display the input flux $Q_{in}$ (red), the leakage flux $Q_l$ (green) and the membrane flux $Q_m$ (blue).

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